7.2. Prescribed Burning

Prescribed burning for hazard reduction has been carried out in indigenous forests in Western Australia for many years. The Forests Department endeavour to burn the entire forested estate every four to eight years. Burning is prescribed when the fuel load reaches 15 tonnes per hectare in jarrah forest types, and six tonnes per hectare in all other forest types.

The computer program, Smoke Advisory Service, is run immediately prior to all proposed prescribed burns to ascertain whether a smoke hazard could be caused to airports or major centres of population. Sample output from this program is presented in Figure 6. If an unacceptable risk of smoke hazard is indicated by the program, the fire may be deferred until more favourable conditions occur.

The fires are ignited using incendiaries of potassium permanganate dropped from aircraft. The Forests Department hires up to two Britten-Norman Islander aircraft for this purpose. One of these aircraft is also used for dieback photography during the autumn.

Prescribed burning is beginning to play a new role in forest management. In addition to hazard reduction, prescribed burning is being used for understorey modification to control the spread of P. cinnamomi.

8. The Growth Model

8.1. Introduction

The principal objective of the interchange was to develop a flexible growth model for natural mixed age stands, which could be applied to indigenous forest stands in Queensland. Because of the extensive data base of white cypress pine (Callitris columellaris F.Muell) held by the Forest Research Branch of the Queensland Department of Forestry, and the relatively simplistic silviculture and stand dynamics of this species, it was decided to develop the model for this species, and later adapt it for other forest types in Queensland.

The model is an elaboration of one developed by the Inventory and Planning Section of the Forests Department for P. radiata and P. pinaster plantations in Western Australia. The W.A. plantation
model is highly successful and is an integral part of the POTS system.

The model is written in ANSI standard FORTRAN and has been run without modification, on C.D.C. Cyber and UNIVAC computers. It is a collection of FORTRAN subroutines which simulate one year of growth in the stand each time the subroutine GROW is invoked. No input or output is performed, as the user is expected to supply interfacing routines to present the information generated in the form that he requires.

Nonlinear equations are extensively used in the model. These equations are generally simple and logical, but the derivation may be quite complex. The rationale for this is to ensure that all functions used in the model have a sound biological basis. Thus pre-determined nonlinear functions are fitted to the data, instead of using linear regression techniques to derive an equation of best fit. This technique is expected to give more reliable results for extreme values, where little data exists.

The following discussion develops the philosophy and rationale of the model. No attempt is made to give a user's guide to the subroutine package. This function will be provided by separate documentation when validation of the model has been completed.

8.2. Overview

The model is designed as a flexible, robust model requiring only stand parameters routinely measured during inventory.

8.2.1. Stand Variables

The primary stand variables derived directly from inventory are the number of trees per hectare within 5 cm dbhob classes, and the site index of the stand.

The secondary stand variables are those derived by the model, from the primary variables, and include the parameters of the diameter distribution, the mean dbhob for each class, the basal area within each class and the stand basal area over bark.

The diameter distribution is derived from the stand table as illustrated in Figure 7. The distribution is continuous and smooth at every point in the range of the diameters, and it conserves relative frequencies in every diameter class. The distribution is totally flexible and there is no requirement that it should
Figure 7
Diameter Distributions derived from Stand Tables

Frequency

Diameter Class

Frequency

Diameter Class
approximate de Liocourt's reverse-J distribution for mixed aged stands (Carroll, 1968), the normal distribution for even aged stands, or any other distribution. The class mean diameters and basal areas are computed by integrating the distribution over the appropriate intervals.

8.2.2. Factors Influencing Growth

The model considers three factors which contribute to determining the growth of an individual tree in the stand. These include the site index and basal area of the stand, and the potential maximum growth of an open grown or dominant tree.

Site index is defined for white cypress pine by an asymptotic diameter height relationship, with the index diameter at 25 cm dbhob (Queensland Department of Forestry, 1981).

The basal area increment of the stand is determined as a function of standing basal area and site index. This increment is proportioned to each diameter class according to its contribution to the stand basal area, subject to the criterion that the class mean increment cannot exceed the potential maximum growth rate.

The potential maximum growth rate is determined by a similar function of dbhob and site index.

8.2.3. The Basal Area and Diameter Increment Functions

The functions for both basal area increment and diameter growth are similar. They are both a direct result of an assumption due to von Bertalanffy (1951), that the growth rate of an organism is proportional to the excess of the energy absorbed by the organism over the energy required to maintain its live tissues.

Both functions are asymptotic, the diameter function asymptotic to a constant, and the asymptote for basal area dependent upon site index. The effect of site index on basal area increment is indicated in Figure 8. The diameter growth trend shown in Figure 9 does not provide a realistic illustration of the diameter growth of trees in a stand as the effects of competition may depress the diameter increment during part of its life. Competition induced reduction in diameter increment is introduced into the model through the basal area increment function.

Site index determines the basal area asymptote, and determines the rate at which a tree diameter approaches its asymptote.
Figure 8
Basal Area Increment

Basal Area Increment (m²/ha/yr)

Basal Area (m²/ha)

High SI
Low SI

Time (years)

High SI
Low SI

Basal Area (m²/ha)
Figure 9
Diameter Increment of Dominant Trees

Diameter Increment

cm/year

DBHOB (cm)

High SI

Low SI

DBHOB cm

High SI

Low SI

Time (Years)
Figure 10

The Distribution of Diameter Increments
For Mean Increments of 1, 2 and 3 cm.

Probability

Mean Increment = 1 cm

Mean Increment = 2 cm

Mean Increment = 3 cm

Diameter Increment
8.2.4. The Distribution of Diameter Increments

The increment assigned to a particular tree within a class is the random Weibull variate with mean equal to the class mean increment. The Weibull distribution is defined between zero and infinity, and thus explicitly excludes the possibility of negative increments. The distribution is characterized by two parameters, one of which can be chosen to ensure that the distribution is almost symmetrical and approaches the normal distribution. The second parameter is determined by the requirement that the distribution mean is equal to the class mean increment.

The distributions for a range of class mean increments are given in Figure 10. It should be noted that the variance of the distribution increases as the mean increases. This is consistent with the heteroscedastic distribution commonly observed for increment data.

8.2.5. Mortality

Individual trees attaining extremely small diameter increments are assumed to die. The probability of mortality in any stem class is computed by integrating the area under the Weibull distribution between zero and the critical diameter increment for survival. This critical diameter increment increases slightly for larger trees.

8.2.6. Recruitment

Estimates of advance growth are computed as a function of basal area. This advance growth is projected for a number of years before the individual stems are recruited into the smallest stem class. Advance growth is assumed to suffer the same rate of mortality as the smallest stem class.

8.2.7. Growth Simulation

Growth of the stand is simulated by the promotion of trees through diameter classes. The proportion of trees promoted from any class is expressed as a probability of movement, computed from the parameters of the diameter distribution and the parameters of the increment distribution.
8.2.8. Implementation

The model is implemented as a collection of FORTRAN routines. The organization of these routines is indicated in Figure 11. A brief summary of each routine follows.

GROW is the usual entry into the model for users. It initializes parameters as required, and invokes other routines to project the stand through one year of growth.

INCD computes the mean diameter increments for each stem class by (1) computing the stand basal area increment, (2) proportioning that increment according to the class contribution to stand basal area, and (3) checking that the resulting class mean diameter increment does not exceed the potential maximum diameter increment.

DEATHS computes the probability of mortality in each stem class.

RECRUT computes recruitment into the smallest stem class.

LIKELY generates a distribution of diameter increments for each class, and applies them to the current diameter distribution to compute probabilities of movement.

MOVES generates the new stand table by promoting trees to the next diameter class.

SPLINE computes the parameters of the new diameter distribution.

MEAND computes the new class mean diameters.

MEANB computes the new class mean basal areas.

Some other routines are available to the user:

MEANH computes the mean height of the trees in each class.

MEANV computes the mean volume in each class.

LIMITS computes the number, basal area and volume of all stems between any two specified dbh db limits.
Figure 11
Generalized Flow Chart of Cypress Model

entry

grow

Yes
initialize only

No

Inc. year
INC D
DEATHS
RECRUT
LIKELY
MOVES

SPLINE
MEAN D
MEAN B

exit
8.3. Potential Diameter Increment

In the development of his growth model, von Bertalanffy (1951) assumed that the rate of growth of the biomass of an organism is proportional to the excess of energy absorbed by the organism over the energy required to maintain its live tissues. For simple organisms, von Bertalanffy assumes that the energy absorbed is proportional to the surface area, and the energy required to maintain the live tissues is proportional to the biomass volume of the organism. Campbell (1981) has generalized these assumptions and extended the method for application to tree growth.

The rate of growth of the biomass volume of a tree is expressed as

\[
\frac{dM}{dt} = u \cdot P \cdot Ls - v \cdot M \tag{1}
\]

where \(u\) and \(v\) are constants of proportion, \(Ls\) denotes leaf surface area, \(M\) denotes biomass volume of the tree, and \(P\) denotes photosynthetic rate, \(0 \leq P \leq 1\).

In this model, this equation will only be applied to stems which are assumed to receive full sunlight, and for these individuals, \(P = 1\).

Substituting the assumed allometric relationships

\[
M = a \cdot D^b
\]

\[
Ls = c \cdot D^d
\]

where \(D\) is diameter (dbhob), the differential equation (1) becomes

\[
\frac{dD}{dt} = \left(\frac{u \cdot c}{a \cdot b}\right) \cdot D^{(b+1) - (v/b)} \tag{2}
\]

A diameter growth function is derived by the integration of (2) over the time period \((t_0, t)\), where \(t\) is an arbitrary age and \(t_0\) is the age at which dbhob \((D)\) is zero:

\[
D = A \cdot \frac{(1 - EXP[-K(t-t0)])}{K} \tag{3}
\]

where

\[
A = \left\{\frac{(u \cdot c)}{(v \cdot a)}\right\}
\]

is the asymptotic diameter of free growing trees,

\[
r = (b-d)
\]

and

\[
K = v \cdot (b-d)/b
\]

are parameters.
The parameter $K$ is assumed to be related to site index,

$$K = p + q \cdot SI$$

so that trees growing on superior sites approach the asymptotic diameter more rapidly than those on inferior sites.

The differential form of (3) provides an estimate of the potential diameter increment of trees in a diameter class for the current year, assuming no competition:

$$I(1) = \frac{dD/dt}{\left(\frac{p+q \cdot SI}{C/D(1)}\right) \cdot \left(\frac{D(1)}{r}\right) \cdot \left(\frac{A/D(1)}{-1}\right)}$$

where $D(1)$ is the mean diameter in the $i$-th class; $I(1)$ is the potential class mean increment, and $SI$ is the site index of the stand.

8.4. Basal Area Increment

A similar argument may be applied to basal area, if all $D$ in equation (4) are replaced with $BA$. However, observing that in biological systems, a greater resource supports a larger population, we also replace $A$ in equation (4) with $f(SI)$, an unspecified function in site index. Thus we have

$$BAI = \left(\frac{p+q \cdot SI}{BA/r}\right) \cdot \left(\frac{f(SI)/BA}{-1}\right)$$

where $BAI$ is the basal area increment, and $BA$ is the stand basal area.

Regression analysis suggests that $q=0$, and $f(SI)=g+h \cdot SI$ for the data set studied during the interchange. Then equation (5) becomes:

$$BAI = \left(p \cdot BA/r\right) \cdot \left(\frac{(g+h \cdot SI)/BA}{-1}\right)$$

8.5. Class Mean Diameter Increment

The actual class mean increment, allowing for competition, may be substantially less than the potential increment predicted by equation (4). To determine the actual class mean increment, the total basal area increment of the stand from equation (6) is proportioned to each of the classes according to that class's contribution to the stand basal area. Should this proportion constitute a diameter increment greater than the potential
increment indicated by equation (4), the increment is reduced accordingly, and the surplus basal area increment is distributed among the other classes.

This is achieved by computing

\[ DI(i) = \text{MIN}[ I(i), D(BA^*, BA(i)/SBA^*) ] \]

(7)

where

- \( DI(i) \) is the actual class mean diameter increment in class \( i \);
- \( I(i) \) is the potential increment from equation (4);
- \( D(b) \) is the class mean diameter increment corresponding to a basal area increment \( b \), in that class;
- \( BA^* \) is the basal area increment from equation (6), adjusted for the increase in classes \( j \), \( j=1, \ldots, N \);
- \( BA(i) \) is the basal area in the \( i \)-th class;
- \( SBA^* \) is stand basal area adjusted for the basal area of classes \( j \), \( j=1, \ldots, N \), ie \( SBA^*=BA(1)+BA(2)+\ldots+BA(1) \).

8.6. Probability of Movement

The derivation of equations 8 to 11 below is taken from Campbell's (1981) unpublished work, and is reproduced here for the benefit of Queensland readers.

8.6.1. Theoretical Background

The probability that a tree in the \( i \)-th diameter class moves into the \((i+1)\)th class after one year of growth can be expressed as a function of the class mean diameter increment, the class interval and the parameters of the distribution of diameters and increments.

Let \( g(y) \) represent a probability distribution defined on the interval \((0, R)\) such that

\[ \int_{0}^{R} g(y) \, dy = G(R) = 1 \]

and let \( m_k \) denote the \( k \)-th moment of \( g(y) \) about the origin.
\[ \int y \cdot g(y) \cdot dy = m \]

Then integrating by parts

\[ m = R \cdot G(R) - k \int y \cdot g(y) \cdot dy \]

and consequently

\[ \int y \cdot g(y) \cdot dy = \frac{(R \cdot G(R) - m)}{k} \]

Now suppose that \( g(y) \) is a distribution defined between zero and infinity. Then equation (8) estimates the value of the integral with negligible error provided that \( R \) is chosen large enough.

8.6.2. Derivation

The probability \( p^*(i) \) that a tree in the \( i \)-th diameter class remains in that class after one year of growth is given by:

\[ p^*(i) = \frac{(1/X) \cdot f(0) \cdot \int_{i}^{i+R+L-D} g(y) \cdot dy \cdot dD}{(i-1)R+L} \]

where \( X \) is the relative frequency in the \( i \)-th class;

\( i \) is the relative frequency in the \( i \)-th class;

\( R \) is the class interval;

\( L \) is the lower limit of the first class containing trees;

\( D \) is a random tree diameter;

\( y \) is a random diameter increment;
f is the distribution of diameters in the i-th class; 

\[ f \]

\[ g \]

is the distribution of increments in the i-th class.

The limits of the leftmost integral are the upper and lower bounds of the i-th diameter class. The limits of the rightmost integral represent the range of permissible increments for a tree of exactly \( D \) centimetres under the conditions that it remains in the i-th class after growth. Therefore the expression for \( p^x(i) \) is the sum, over all permissible values of \( D \), of the probability that a tree is in, and remains in the i-th class, and is exactly \( D \) centimetres diameter before growth.

The diameter distribution \( f(i) \) is identified with the quadratic function describing the diameter distribution:

\[ f = \frac{1}{RN} \left( A \left( \frac{(D-L)}{RN} \right)^2 + B \left( \frac{(D-L)}{RN} \right) + C \right) \]

such that

\[ \frac{(i-1)}{N} \leq \frac{(D-L)}{RN} \leq \frac{i}{N} \]

\[ f \]

\[ (i-1)R+L \]

\[ f(D) \cdot dD = x \]

\[ (i-1)R+L \]

where \( N \) is the number of diameter classes containing trees and the remaining symbols have been defined previously.

Implementing the rightmost integral defined in (9) and introducing the transformation of variables \( z = iR+L-D \), \( p^x(i) \) is expressed:

\[ R \]

\[ p^x = \frac{1}{(X \cdot R \cdot N)} \int \left( A \left( \frac{(iR-z)}{RN} \right)^2 + B \left( \frac{(iR-z)}{RN} \right) + C \right) \cdot G(z) \cdot dz \]

(10)

where \( G(1) \) is the cumulative distribution of increments in the i-th class.

The integral (10) is evaluated using the relations (8). The evaluation may be exact for certain choices of \( g(i) \) but, in general, is an approximation. It was stated above that the approximation leads to negligible error provided that \( R \) is chosen large enough. \( R \) is now identified with the diameter class interval, and this condition can be interpreted as requiring that
the diameter class interval should be large relative to the expected increments.

The substitution of the relations (8) into the integral (10) yields:

\[
p^\ast(i) = \frac{1}{X} \left( A \frac{R - m3}{(3 \cdot R \cdot N)} \right) + 3 \frac{2}{3} + 3 \\
+ \left[ 2A \cdot (1/N) \cdot B \cdot (R - m2)/(2 \cdot R \cdot N) \right] \frac{2}{4} + \frac{2}{4}
\]

Provided that the class interval is set large enough to ensure that trees may not move through more than one diameter class in one period of growth then the probability (p(i)) that a tree in the i-th class moves into the next class after growth is:

\[
p(i) = 1 - p^\ast(i)
\]

\[
= \frac{1}{X} \left( A \cdot m3/(3 \cdot R \cdot N) \right) + \frac{3}{3} + \frac{3}{3}
\]

\[
- \left[ 2A \cdot (1/N) \cdot B \cdot m2/(2 \cdot R \cdot N) \right] \frac{2}{4} + \frac{2}{4}
\]

\[
+ \left[ A \cdot (1/N) \cdot B \cdot (1/N) + C \cdot m1/(R \cdot N) \right] \frac{2}{4} + \frac{2}{4}
\]

(11)

The distribution of increments (g(i)) is identified with the Weibull distribution with parameters fixed as follows:

\[
c = 3.6
\]

\[
g(i) = \left( \frac{c}{b(i)} \right) \cdot (y/b(i))^{c-1} \cdot \exp\left( -(y/b(i))^{c} \right) 
\]

(12)

\[
b(i) = I(i)/\text{GAMMA}(1/c+1) = 1.1098 \cdot I(i)
\]

where I(i) is the mean increment for the i-th class computed from equation (7), and GAMMA represents the gamma function.
With this choice of parameters, the distribution of increments \( g(i) \) has the following properties:

1. It is almost symmetrical (for \( c = 3.6 \));

2. Its mean \( m_1 \) is equal to the class mean diameter increment \( I(i) \);

\[
I(i) \times 1.0951;
\]

3. \( m_3 = I(i) \times 1.2844 \).

3.6.3. Comparison with Davis' Formula

Methods more commonly used for promoting trees through diameter classes are discussed by Davis (1966). The formulae recommended by Davis assume a uniform distribution of trees within diameter classes. In order to compare probabilities of movement predicted by Davis' and the above formulae, it is necessary to fix the values of some parameters in (11):

\[
N = 1, \quad X = 1, \quad A = B = 0, \quad \text{and} \quad C = 1.
\]

When these values are substituted in (11), the expression simplifies to:

\[
p(i) = m_1/R = I(i)/R
\]

which is the classical formula recommended by Davis. Thus equation (11) may be regarded as a generalization of Davis' formula.

Table 3 presents a comparison between the probabilities produced by (11) against the corresponding probabilities produced by Davis' formula. The calculations refer to a stand in which all the trees are concentrated within a single 5 cm dbh ob class.
### Table 3.

<table>
<thead>
<tr>
<th>Mean Increment</th>
<th>Computed Probabilities</th>
<th>% Difference</th>
<th>( \frac{(a-b)}{b} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Davis (^{(a)})</td>
<td>Eqn(11) (^{(b)})</td>
<td></td>
</tr>
<tr>
<td>1 cm</td>
<td>0.125</td>
<td>0.046</td>
<td>+171.7%</td>
</tr>
<tr>
<td>2 cm</td>
<td>0.250</td>
<td>0.165</td>
<td>+51.5%</td>
</tr>
<tr>
<td>3 cm</td>
<td>0.375</td>
<td>0.327</td>
<td>+14.6%</td>
</tr>
<tr>
<td>4 cm</td>
<td>0.500</td>
<td>0.500</td>
<td>0.0</td>
</tr>
</tbody>
</table>

8.7. Promotion into the Next Class

Let the vectors \( n \) and \( n' \) represent respectively the stand table before and after growth. Then

\[
n' = P(j,k) \cdot n
\]

where \( P(j,k) \) is the stochastic matrix with entries defined as follows:

\[
\begin{align*}
P(i,i) &= 1 - p(i) \\
P(i+1,i) &= p(i) \\
P(j,i) &= 0, \text{ (all } j \text{ except } j=i \text{ and } j=i+1), \text{ and} \\
p(i) &= \text{the probability returned by equation (11).}
\end{align*}
\]

After executing the operations described above, the largest and smallest diameter classes are tested to confirm that they contain more than a specified small proportion of the total stocking. If either of the classes contains less than that proportion, it is eliminated by promoting or demoting trees into a neighbouring class. This prevents unnecessary fragmentation of the stand.

8.8. Mortality

The mortality function assumes that if the increment of any tree over a period is less than a certain critical increment defined by a function

\[
M = a + b \cdot D \tag{13}
\]

then that tree will die. Equation (13) requires that for a larger tree to survive, it must achieve a higher increment than a smaller tree.

The probability of mortality occurring within any class is
determined as the probability of increments less than the critical increment, M, occurring within that class. This is computed from the integral of the Weibull distribution:

\[
P = \int_{0}^{\lambda} \frac{c-1}{c} \left(\frac{y}{b(i)}\right)^{c-1} \left(1 - \exp\left(-\frac{y}{b(i)}\right)\right)^{c} \, dy
\]

where \(c\) and \(b(i)\) are defined for (12) above.

8.9. Recruitment

Recruitment is predicted by determining regeneration as a function in stand basal area and site index, and projecting this regeneration for a number of years before recruiting it into the smallest diameter class.

For the data of this study, site index was found to contribute little, and regeneration is determined as

\[
R = \exp\left(a + b \cdot SBA\right) - 1
\]

This regeneration is projected for a number of years until it reaches breast height (1.3 m), when it is recruited into the smallest class. Each year it is subjected to the same probability of mortality as is computed for the smallest diameter class containing trees.

8.10. Stand Diameter Distribution

The stand table, which is input as a histogram, is transformed to a quadratic spline function which is continuous and smooth at every point in its range and which conserves the relative frequency in every diameter class interval (Campbell, 1981).

The spline function is a set of quadratic functions of the form:
\[ f(x) = A x^2 + B x + C \quad \frac{(i-1)N}{x} \leq iN \]

where
\[ x = \frac{(D - 0)}{(R \cdot N)} = \text{relative diameter} \]
\[ N = \text{the number of diameter classes spanned by the distribution} \]
\[ D = \text{tree dbh} \]
\[ D = \text{the lower limit of the least class} \]
\[ R = \text{class interval} \]
\[ A, B, C \text{ are parameters to be determined} \]

Let \( X(i) \) denote the relative frequency of trees in the \( i \)-th class and impose the conditions stated above:

1. \( f(i/N) = f(i/N) \quad \text{(continuous)} \)
   \[ i \quad i+1 \]

2. \( f'(i/N) = f'(i/N) \quad \text{(smooth)} \)
   \[ i \quad i+1 \]

3. \[ \frac{f(x)}{dx} = X(i) \quad \text{(conserves relative frequencies)} \]
   \[ \frac{(i-1)}{N} \]

These three conditions imply \( 3N-2 \) conditions. Two additional conditions are required to fully specify the spline functions.

4. \( f(0) = 0 \)
   \[ 1 \]

5. \( f(N) = 0 \)
   \[ N \]

are convenient choices which allow simplifications in determining the parameters of the spline function using a method due to Campbell (1981). However, it is necessary to create a dummy class with dbh less than zero if there are trees in the 0-5 cm dbh class.
To determine the parameters, let $P(i)$ and $Q(i)$ denote sequences defined recursively as follows:

\[
\begin{align*}
P(1) &= 1 \\
Q(1) &= 1 \\
P(i+1) &= 2 \cdot Q(1) + P(1) \\
Q(i+1) &= Q(i) + P(i+1)
\end{align*}
\]

so that:

\[
\begin{align*}
P(i) &= (1, 3, 11, 41, 153, \ldots) \\
Q(i) &= (1, 4, 15, 56, 209, \ldots)
\end{align*}
\]

Set:

\[
\begin{align*}
A &= 0 \\
B &= 2 \cdot (-1)^{i-1} \\
C &= 0
\end{align*}
\]

The parameters $A$, $B$, $C$ $(i = 1, N)$ are determined by successive applications of the relations:

\[
\begin{align*}
A_{i+1} &= A_i + 3N \cdot (X_{i+1} - Y_{i+1}) \\
B_{i+1} &= B_i - 2N \cdot 3i (X_{i+1} - Y_{i+1}) \\
C_{i+1} &= C_i + N \cdot 3i \cdot (X_{i+1} - Y_{i+1})
\end{align*}
\]

$Y(i+1)$ is the integral of the $i$-th function over the $(i+1)$th interval.
\[
\frac{(i+1)}{N} \int_1^i f(x) \, dx
\]
\[
= \frac{A}{3} \left[ (i+1) - i \right] + B \left[ (i+1) - i \right] + C \frac{1}{N}
\]

The function generated by this procedure is not always a distribution as negative values may occur within its range. If negative values are detected within any class, a linear or constant function may be substituted for the quadratic spline within that class.

This problem only arises when the relative frequency within a class is small relative to adjacent classes. Therefore, the inconsistencies introduced by substituting the linear function are negligible.

8.11. Class Mean Diameters

The mean diameter of trees in the \( i \)-th diameter class is:

\[
\frac{1}{N} \int_1^{i-1} x \cdot f(x) \, dx + \frac{L + R_i \cdot N}{x(i)}
\]

where \( f(x) = A x + B x + C \) is the diameter distribution

\[
x = \frac{(D-L)/(R_i \cdot N)}{x(i)} = \text{relative diameter}
\]

\[
D = \text{tree dbhob}
\]

\[
L = \text{lower limit of least class}
\]

\[
R = \text{class interval}
\]

\[
N = \text{number of classes containing trees}
\]

\[
x(1) = \text{relative frequency of trees in the } i\text{-th class}
\]

The overall stand diameter can be obtained by summing the product of the relative frequencies and class mean diameters for all classes containing trees.

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8.12. **Class Mean Basal Area**

The basal area in any dbhob class is

\[
B = \frac{1}{N} \int_{(i-1)/N}^{i/N} (L + x \cdot R \cdot N) \cdot f(x) \cdot dx
\]

\[
(15)
\]

where \( f(x) = A \cdot x^2 + B \cdot x + C \)

\[
x = \frac{(D-L)/(R \cdot N)}{Y} = \text{relative diameter}
\]

\[Y = \text{total stocking}\]

\(D, L, N\) and \(R\) as previously defined for (14).

8.13. **Class Mean Heights**

Mean heights for each diameter class are computed from Henry's Mitscherlich equation, using class mean diameter and site index:

\[
H(i) = k - (k-1.3) \cdot \left( \frac{(k-SI)/(k-1.3)}{25} \right)
\]

where \( k = a + b \cdot SI\), with \(a\) and \(b\) constants.

The index dbhob of the curve is 25 cm, and thus the height of a 25 cm dbhob tree is equal to the site index. This curve is illustrated in Figure 12.

8.14. **Class Mean Volumes**

Volumes are computed using an equation developed by N.B. Henry (Queensland Department of Forestry, 1979):

\[
V = a + b \cdot A + c \cdot H + d \cdot A \cdot H
\]

where \(A\) is the basal area of the tree, and \(H\) is the average height of a 25 cm dbhob tree, that is, the site index of the stand.

This equation may also be written

\[
V = a \cdot S + b \cdot A^2 + c \cdot S \cdot H + d \cdot A \cdot H
\]

(16)
where $A^*$ is the basal area, and $S$ is the number of trees, either between any specified limits, or in the stand.

8.15. Proportion of Stand between Specified Limits

It is often required to determine the stocking and basal areas between specified limits not aligned with the diameter classes. For example, a commonly used dbhob limit used in cypress pine is 19 cm dbhob.

The number of stems between any specified limits may be determined by

$$S = T_0 (A (u - b) / 3 + B (u - b) / 2 + C (u - b))$$

where

- $T_0$ is total stocking
- $A$, $B$, $C$ are parameters of the diameter distribution
- $u = (D - L)/(R \cdot N)$
- $b = (D - L)/(R \cdot N)$
- $D$ and $L$ are the upper and lower dbhob limits

Basal areas between the specified limits may be determined by application of equation (15), and volumes may be computed by substituting the numbers and basal areas determined above, into equation (16).

8.16. Further Refinement

The model has been developed to a workable state, and shows promise in giving reliable results. However, a number of untested assumptions were made in its development, and some deficiencies were evident in the data base. The following discussion highlights the deficiencies and the assumptions which require further
development.

8.16.1. Site Index

N. B. Henry's Mitscherlich site index equation shows great promise as a descriptor of increment in cypress stands. However, there is some evidence that the estimate of site index may be affected by stand treatment and logging operations. Further work needs to be carried out to determine methods of calculating an estimate of site index free from such variation.

8.16.2. Basal Area Increment

Because of deficiencies in the data base, the upper asymptote for the basal area function is poorly defined, and this may have profound effects when modelling extreme silvicultural treatments. Data from plots of higher basal areas than were available at the time of the interchange have been located, and a revised function is being computed.

8.16.3. Diameter Growth

Although the asymptotic diameter for cypress has been estimated at 100 cm, no measurements of increments exist for stems greater than 55 cm dbhob. To redress this deficiency, stem analysis of some larger cypress stems is being carried out in an attempt to secure further data. If this method shows promise, further stem analyses will be performed and a revised equation will be computed.

8.16.4. Mortality

The mortality function used in the model was conceived and developed by the author. Although it appears to work well, it is difficult to determine the critical increments, below which mortality occurs. The parameters in the function were initially determined by an educated guess, and refined by trial and error.

A mortality function advocated by Buchman (1979) overcomes these problems, and could be incorporated into the model. Buchman's function should be evaluated for cypress pine, and if suitable, should be included in the model.
8.16.5. Recruitment

The recruitment function in the model is based on very little data, and the treatment of advance growth is, at best, conjecture. Observations and measurements should be made on the behavior of cypress seedlings to 1.3 metres height, so that this function can be improved as necessary.

8.16.6. Distribution of Increment

Further work needs to be carried out to verify the assumption that the fraction of basal area increment in any diameter class is directly proportional to the fraction of the stand basal area in that class.

8.16.7. Other Components of the Stand

The model was developed for stands which are essentially pure cypress. The effect of other species in the stand should be quantified, so that the model can be extended to stands with a larger component of other species.

8.17. Implications of the Model

Analysis of silvicultural options for cypress pine has not yet been performed, so no recommendations for the management of the resource can be made. However, some important observations regarding the data base should be made.

Although the Forest Research Branch holds an enormous data base for cypress pine, much larger than is required for effective analysis, the data is concentrated into a small range of stand structures. There is little data for plots having high basal areas, few plots with sufficiently low stocking to allow growth without competition, and few plots where competition induced mortality is allowed to manifest itself.

If an effective model is to be developed, which will enable all silvicultural possibilities to be explored adequately, these extreme situations need to be included in the data base. The Forest Research Branch should endeavour to redress this imbalance in the data base, not only for cypress pine, but for all forest types of commercial interest.
References


9. Benefits of the Interchange accruing to W.A.

During the interchange, many officers of the Forests Department debated with the author, differences in policy and practice between the Forests Department of Western Australia and the Queensland Department of Forestry. Of particular interest to many of the staff was the Forester Tenure system recently proposed in Queensland.

The extension officer of the Forests Department expressed particular interest in the successful Forest Open Days which have been held in Queensland for some years. As a direct result of the discussions held during the interchange, the Forests Department intend to hold their first Forest Open Day later this year.